RECENT RESEARCHES IN THE STRUCTURE OF THE UNIVERSE.[1]

II.

Localisation of the Stars in Space by a Sorting Process.

THE method may be best explained as a *sorting* process. The process was not actually followed; it would have been too laborious, and would have met with some difficulty.[2] But the difference is immaterial, and the present description has, I think, the advantage in point of clearness. Let each of the stars of the second, third, &c., to the eighth magnitudes be represented by a little card on which are inscribed the apparent magnitude and the apparent proper motion of the star. Then imagine three sets of boxes.

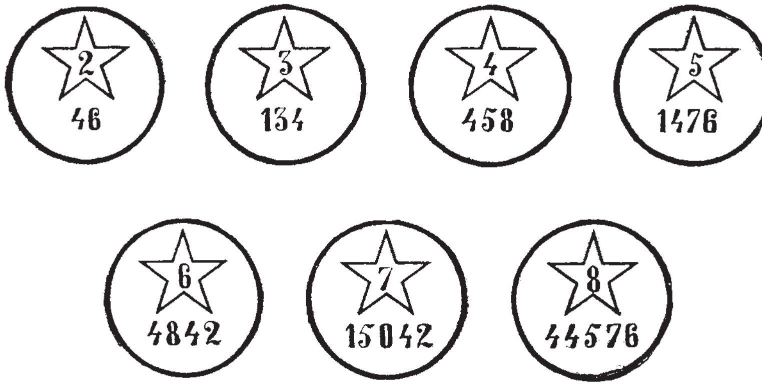

Fig. 1.

Classification according to Magnitude.

1st *Set.—Apparent magnitude boxes* represented in Fig. 1.—In the box for the second apparent magnitude, as many cards are put as there are stars of the second magnitude in the sky. The total numbers of stars for each magnitude are inscribed on the lid. We thus see that there are in the whole of the sky forty-six stars of the second magnitude, 134 of the third, and so on.

According to Magnitude and Proper Motion.

2nd *Set.—Magnitude-motion boxes* (Fig. 3). The stars in each of the former series of boxes are re-distributed over a series of boxes, each of them containing stars of a determined apparent motion. By way of an example, Fig. 3 shows this new classification for the stars of the *fifth* apparent magnitude. There is, of course, another such series for each one of the apparent magnitudes. Those for the *fifth* have been distributed over twenty-eight new boxes. In the first have been collected the cards representing the stars with a proper motion of 0″ to 1″ per century. The average motion is 0·5, and this has been inscribed on the lid. The little arrow indicates that this number represents a motion. The number 5 surrounded by a star refers to the fact that we have exclusively to do with stars of the fifth apparent magnitude. The second box contains the stars with proper motion between 1″ and 2″ per century, &c. For the larger motions the limits have been taken somewhat wider. In the eleventh box the motions 10″ to 15″ are contained, in the thirteenth those between 20″ and 30″, and so on. The number of star-cards in each box has been inscribed on the lower right-hand corner of the lid. The figure thus shows, for instance, that there are in the sky ninety stars of the fifth magnitude having a proper motion between 0″ and 1″ per century. We have thus arranged the stars according to both the rough criteria of distance at our disposal; for we know perfectly well that in a very general way the fainter the stars and the smaller their apparent motion the further they must be away.

For each of the groups thus obtained we are now able, according to what has been said before, to derive the *mean* distance. This determination being made, we obtain the mean distances expressed in light-years which have been inscribed on the lid with the letter MD prefixed. Already we may see now how incorrect it is to imagine all the stars of the fifth magnitude to be placed at one and the same distance, as Struve did. According to the numbers in our figure, the distance varies from 1670 light-years for the stars of the first box to eleven light-years for those of the last. It is true that just the data for these extreme boxes are the most uncertain; still, it is evident that even in these mean distances there must be an enormous range.

But to proceed. The eighty-six stars in our sixth box (see Fig. 3) are at an average distance of 248 light-years.

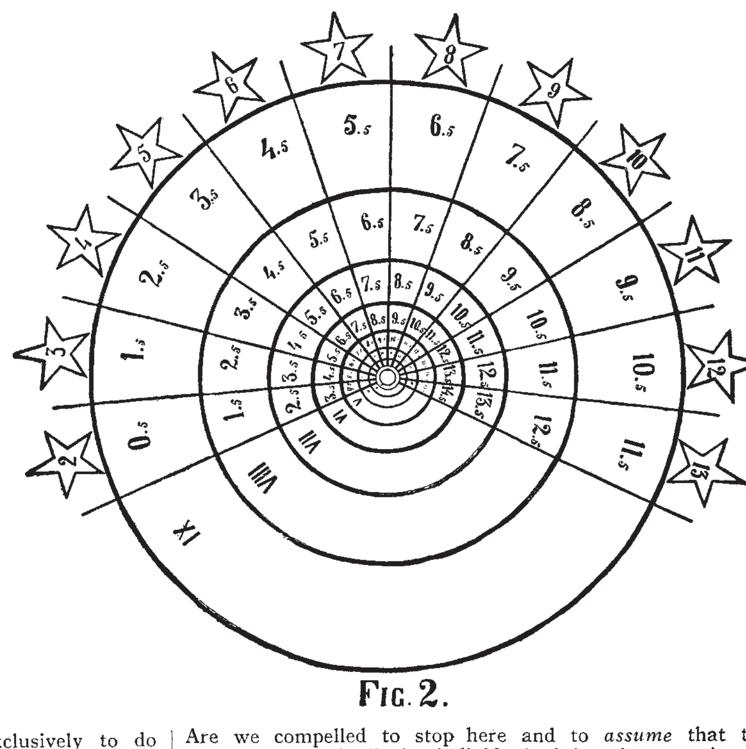

FIG. 2.

Are we compelled to stop here and to *assume* that the real distance of *all* the individual eighty-six stars is 248 light-years? If it were so we would *surely* still have gained a considerable advantage over Struve. For, owing to want of other data, he saw himself compelled to treat all the stars of the fifth magnitude, that is, the whole of the twenty-eight groups in our boxes, as if they were all at the mean distance of the whole. But yet there would remain in our solution a defect *of the same kind*, and it

[1] Discourse delivered at the Royal Institution on Friday, May 22, by Prof. J. C. Kapteyn. Continued from p. 212.

[2] For many of the stars used the proper motion is still not known. What is known, however, is the percentage of the stars of each magnitude having a determined proper motion. This knowledge enables us to put in every box the required *number* of cards showing a determined proper motion, and this is all that is wanted in what follows.

would be impossible to say in how far the results definitively to be obtained would be influenced. Happily there is an escape.

For our last classification, the classification in the distance-boxes, it is of no particular advantage that every individual star gets in its proper distance-box. It will be sufficient to know how many stars will finally be found in each distance-box. If this result is obtained, we shall presently see how easy it becomes to study the problem put at the beginning of this lecture. Our aim will be evidently reached if we can find out *how many per cent.* of the stars in any one box have such and such a distance. Now, in order to determine these percentages, it will be sufficient to investigate *a sample* of our stars.

Stars of Measured Distance taken as a Sample.

Happily there is the possibility of taking a sample that will help us out of the difficulty, for, as we know, there are in the sky a hundred stars of which astronomers have succeeded in determining the individual distance with some accuracy. We take these as our sample. They are distributed over a great many of our boxes.

We take them all out, having a care to note for all of them the mean distance of the stars in the box to which they belong. For all the hundred stars we now compare their mean distances to their true distances, and thus find out how many per cent. of them have true distances between *two* and *three, four* and *five tenths*, and so on, of the mean distance.

3rd Set.—Distance boxes. These percentages are all we want for our last distribution, the distribution over the distances. It is true that our sample is a somewhat undesirably small fraction of the whole; it shows besides some other weak points, but it appears happily *a posteriori* that even rather considerable uncertainties in these percentages have but an unimportant influence on the results. We are thus at last enabled to distribute our star-cards according to the true distances. I made the distribution over the spherical shells shown in Fig. 2.

The dimensions of these shells have been so chosen that if a star is removed from one shell to the next further one, the observer at the centre will see the star grow fainter by just one magnitude, that is, it will grow very nearly $2\frac{1}{2}$ times fainter.

The figure is not well fitted for bringing out the details of our results. The shells become too narrow towards the centre, and the more central ones do not allow of the insertion of sufficiently clear figures. For this reason I constructed Fig. 4. The numbers valid for the several spherical shells have here been entered in equally broad horizontal rows. The drawing does not therefore show the real dimensions, but these as expressed in light-years, which may be read off on the right-hand side of the drawing. We thus see that the central sphere extends to a distance of twenty-one light-years, that the second spherical shell extends from twenty-one to thirty-three years, and so on. In these rows a last set of boxes is placed. There is a box for each apparent magnitude in each of the rows. The stars of the boxes of Fig. 3 are thus, of course, all contained in the vertical row of boxes, corresponding to apparent magnitude five in Fig. 4.

Distribution according to Distance Illustrated by Example.

In order to illustrate by an example how the stars of the boxes in our Fig. 3 are distributed over our different shells, that is, over our *distance boxes* of Fig. 4, take the seventh box. It contains seventy-seven stars at a mean distance of 220 light-years. Our countings on the sample showed that about *one-fifth* of the stars have *true* distances which are between 37 per cent. and 59 per cent. of their *mean distance* (derived from their apparent magnitude and proper motion). Therefore about one-fifth of our seventy-seven stars must have true distances between 37 per cent. and 59 per cent. of 220 light-years, that is, between eighty-two and 130 light-years—or, finally, fifteen stars of our box must find their place in the fifth shell of Fig. 4, that is, in the box corresponding to the fifth apparent magnitude in that shell. In precisely the same way I find that twenty-one of them must be placed in the sixth shell, eighteen in the seventh, ten in the eighth, and so on.

If, after that, we repeat the process for all the remaining boxes of Fig. 3, we get, for the fifth apparent magnitude, the numbers inscribed on the lower side of the boxes corresponding to that magnitude in Fig. 4.

Further than for the eleventh shell no numbers have been entered. They become too uncertain. As, however, we know the *total* number of stars of each apparent magnitude, we know the aggregate number which remains to be distributed over the whole of the further shells.

What has here been explained for the stars of the fifth magnitude has been also done for the other magnitudes between the second and the eighth. The whole of the results are shown in our Fig. 4.

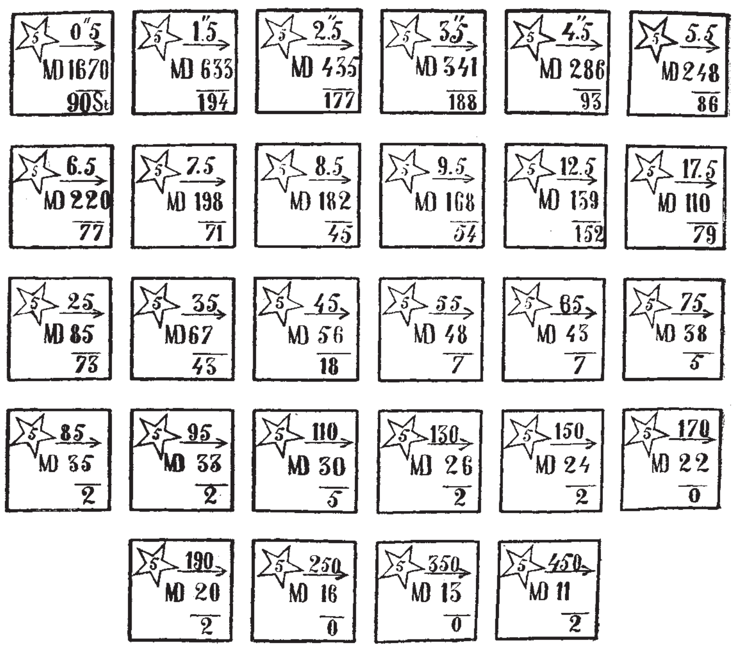

Fig. 3.

Stars of Equal Luminosity brought together.

The main result of the investigation is embodied in these numbers—and *first*, in every box stars have now been brought together of equal absolute magnitude—that is, of equal luminosity. For as the stars in each box are at the same distance, and as, at the same time, they are of equal *apparent* brightness, they must, of necessity, be of equal total light-power, that is, according to our definition, of equal luminosity or absolute magnitude. For the absolute magnitude of a star I have taken the magnitude the star would show if placed at a distance of 326 light-years. The choice of just this number is simply a matter of convenience, and need not be explained here.

As a consequence, the stars at a distance of 326 years, which to us appear as stars of the fifth magnitude, will have also the absolute magnitude five. Those of the same apparent magnitude, but at a distance of 517 light-years—that is, just one shell further—must have the absolute magnitude four in order to show us the same brightness, notwithstanding the greater distance. Now our eighth shell lies just between these limits of distance. In the middle of this shell, therefore, the stars of apparent magni-

tude five must have absolute magnitude 4·5. In the box, therefore, belonging to the fifth apparent magnitude, eighth shell, all the stars are of absolute magnitude 4·5. In the ninth shell a star must already have the absolute magnitude 3·5 in order to shine as a fifth apparent magnitude at this greater distance, and so on. In this way the absolute magnitudes were found which in our figure have been inscribed on the lids of the boxes.

We are now able to derive at once the *mixture law*, *i.e.* the proportions in which stars of different absolute magnitude are mixed in the universe. For in one and the same shell (eleventh) we find two stars of absolute magnitude −1·5, as against three of magnitude −0·5, fifteen of absolute magnitude 0·5, seventy-six of absolute magnitude 1·5, &c.

That is, our results for the eleventh shell furnish us with the proportion in which stars of absolute magnitude −1·5, −0·5, &c., to 4·5, are mixed in space. The tenth shell gives the proportions for all the absolute magnitudes between −0·5 and 5·5, and so for the rest. All the shells together give the proportions for the absolute magnitudes

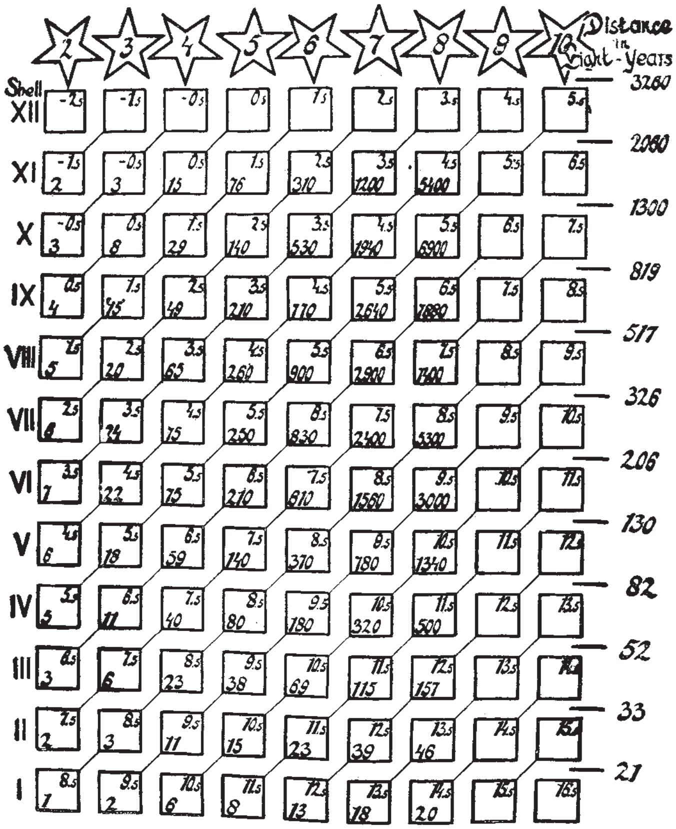

Fig. 4.

−1·5 to 14·5, that is, for a range of not less than sixteen magnitudes. Not only that, but most of the proportions are determined independently by the data of quite a number of shells. So, for instance, the proportion of the stars of absolute magnitude 4·5 to those of absolute magnitude 5·5. Each of the six shells from the fifth to the tenth furnishes a determination of this proportion. All of them are not equally trustworthy. If we take this into account, we find that the agreement of the several determinations is fairly satisfactory. By a careful combination of all the results, a table representing the law of the mixture of the stars of different absolute magnitude was finally obtained. Rather than show you the direct result, however, I will first replace the absolute magnitudes by luminosities expressed in the total light of our sun as a unit. This will have the advantage of presenting a more vivid image of the real meaning of our numbers.

By photometric measures it was found that the sun, placed at a distance of 326 light-years, would shine as a star of magnitude 10·5. In other words, the sun's absolute magnitude is 10·5. A star of absolute magnitude 9·5 will, therefore, have 2·5 times the light-power—that is, 2·5 times the *luminosity* of the sun. A star of absolute magnitude 8·5 will again have a luminosity which is 2·5 times greater, and so on.

Such results evidently enable us to transform our absolute magnitudes into luminosities. Thus translated, I found the results shown in the following table.

Luminosity Table.

Within a sphere having a radius of 555 light-years, there must exist :—

1 star	10,000	to 100,000	times more luminous than the sun
46 stars	1,000	,, 10,000	,, ,, ,, ,,
1,300 ,,	100	,, 1,000	,, ,, ,, ,,
22,000 ,,	10	,, 100	,, ,, ,, ,,
140,000 ,,	1	,, 10	,, ,, ,, ,,
430,000 ,,	0·1	,, 1	,, ,, ,, ,,
650,000 ,,	0·01	,, 0·1	,, ,, ,, ,,

This table represents what, up to the present time, we know about the mixture law.

The fainter the stars, the more numerous.

The rate at which the numbers increase with the faintness is particularly noticeable for the very bright stars.

Passing to the fainter stars, this rate gradually diminishes, and it looks as if we must expect no further increase in number for stars the luminosity of which falls below one-hundredth of that of the sun. Meanwhile, this is simply a surmise. For stars of this order of faintness data begin to fail. Here, as in nearly every investigation about the structure of the stellar system, the want of data for stars below the ninth apparent magnitude makes itself very painfully felt.

But let us come back to our Fig. 4. I will *first* remark that, knowing the mixture law, we can predict the number of stars that we shall get in the empty boxes belonging to the ninth, tenth, &c., magnitude, as soon as continued astronomical observations will permit us to include these stars in our discussion. For the mixture law, as derived just now, shows that in our universe the stars of absolute magnitude 5·5 are 3·5 times as numerous as the stars of absolute magnitude 4·5.

Now as in the eleventh shell the number of stars of the absolute magnitude 4·5 is 5400 (see Fig. 4), there must be 3·5 times 5400, that is, 18,900 stars of absolute magnitude 5·5 in this shell. These belong all in the box of the ninth apparent magnitude of this shell. In the same way we obtain the number of stars to be expected in the boxes of the tenth, eleventh, &c., apparent magnitude for all our shells down to the eleventh. There is exception only for the boxes belonging to the lower shells, for which the absolute magnitude would exceed 14·5.

It is evident, however, that the number of stars in these exceptional boxes must be small, and for what follows they are of little importance.

Star-density.

In the second place, our boxes now also lead to the determination of the *star-densities*. For the volumes of the consecutive shells are perfectly known; they are in the proportion of 1 : 3·98. For the sake of convenience, let us say that the volume of each shell is exactly four times that of the next preceding one. Now, to take an example of the determination of the densities, consider the ninth and tenth shells (see Fig. 4). In the ninth there are forty-nine stars of absolute magnitude 2·5. Therefore, if in the tenth the stars were as thickly crowded as in the ninth, there would occur in this shell four times forty-nine, that is 196 stars of this absolute magnitude 2·5.

In reality we find but 140 of these stars. The conclusion evidently must be that the star-density in the tenth shell is about 140/196, that is, about two-thirds of that in the ninth shell. A similar conclusion is obtained by

comparing the number of the stars of absolute magnitude 3·5 in the two shells. The values obtained from the magnitudes 0·5 and 1·5 may be neglected. Owing to the exceedingly small number of stars, they must necessarily lead to untrustworthy results. From all the rest I found that the density in the tenth shell must be about 64 per cent. of that in the ninth shell. The proportion between the densities in the other shells was determined in exactly the same way.

A slight defect in our results was then discovered. We should exceed the limits of the time allowed for this lecture by entering into a consideration of this defect. It must be sufficient to state that it was not difficult to remove it. After that it appeared that the density in the first six of our shells is nearly the same. The density in these shells, that is, in the neighbourhood of our sun, is such that about 2000 stars of a luminosity exceeding one-hundredth that of the sun must be contained in *a cubic light-century*. After the sixth shell the density diminishes gradually at such a rate that in the eleventh shell the density has fallen to about 30 per cent. of what it is in the vicinity of the solar system.

In what precedes we tried to give a solution of the problem put at the beginning of this lecture—a solution, however, which embraces only that part of the universe which is contained within a distance of about 2000 light-years from our solar system. Is there no possibility of getting beyond this distance?

I think there is, but, of course, you will not be astonished to find that the certainty of our conclusion diminishes as we get deeper and deeper into the abysses of space.

One of the reasons why the method thus far applied breaks down beyond the eleventh shell is that our data about proper motion are not refined enough to determine this motion with sufficient accuracy as soon as it is below 1″ in a century. Even the somewhat greater motions are rather uncertain. The proper motions thus cannot help us much beyond a certain distance. But we have still one valuable element for the solution of our problem. This element is the total number of stars separately for the apparent magnitudes. Thanks mainly to the photometrical researches at the Harvard Observatory, it has become possible to determine with considerable accuracy the total number of stars of the first, second, &c., to the eleventh magnitude; with a fair degree of accuracy even those for the magnitudes down to the fourteenth (inclusive).

The density in the shells beyond the eleventh, not only for the stars down to the eighth apparent magnitude, but, according to what has been said a moment ago, also for the apparent magnitudes of nine, ten, &c., to fourteen, has to be determined in such a way that the addition of all the numbers in any one vertical column of Fig. 4 produces just these totals for the corresponding apparent magnitudes.

It can be proved that after the eleventh shell the density must, on the whole, continue to diminish. If we assume that this diminution is gradual and proportional to the increase in distance, it becomes very easy to determine the rate of this diminution, and consequently the distance at which the density becomes *zero*, that is, the distance at which we reach the limit of the stellar system. We cannot enter into fuller particulars here. It must be sufficient to say that in this way we are led to conclude that the further diminution of density must be slow, so slow that in the assumption made above the limit of the system is only reached at a distance of some 30,000 light-years.

Hypotheses Underlying the Results.

In conclusion, a few words on the question, In how far are the results now obtained to be considered as established?

The answer must be, They can be considered to be established only in so far, and no further, than we can trust the truth of the hypotheses which still underlie our reasoning.

For future consideration there thus remains the question, In how far can we test the validity of these hypotheses?

These hypotheses are the following:—

(1) The mixture was assumed to be the same at greater and smaller distances from the solar system.

(2) The same was done for different distances from the galaxy.

(3) The universe was assumed to be transparent, that is, it was assumed that the absorption of light in space is *zero*.

Can we get rid of these hypothetical elements?

I think we can, at least to a very great extent.

As to the *first*. Our Fig. 4 already goes far in enabling us to judge whether it is true or not. For evidently both our sixth and our ninth shell give the nature of the mixture, at least of the stars of absolute magnitude 3·5 to 6·5. Therefore, so far as these stars are concerned, we are able to see whether or not the mixture is the same at the distance of 650 light-years as it is at the distance of 170 light-years. Likewise, the figure enables us to make the comparison in other cases. As soon as we possess the necessary data for a longer range of apparent magnitudes, say down to the fourteenth or fifteenth, we shall be able to dispense to a very large extent with our first hypothesis.

As to the second, the possible variation of the mixture with the distance from the Milky Way, it is largely only the question of treating the stars in different galactic latitudes separately. So far as I can see, there are no particular difficulties in the way of such a separate treatment, at least not since the nature of certain anomalies in the distribution of stellar motions has been elucidated.

Absorption of Light in Space.

Last, not least. Is the universe really absolutely transparent? There are reasons which make this seem very doubtful. A couple of years ago I obtained some evidence in the matter which shows that the absorption of light in space, if it exists to an appreciable amount, must at least be so small that over a distance of a hundred light-years not more than a few per cent. of the light can be lost. To determine so small an amount to within a small fraction of its total value will be a difficult task indeed. Still, we can even now see definite ways, which, given the necessary data for very faint stars and nebulæ, will probably enable us to overcome this last difficulty.

This want of data for very faint stars, which, in the present investigation, makes itself felt at every step, has led a number of astronomers to concerted action.

The express purpose of their cooperation is to collect data of every kind for stars down to the faintest that can practically be reached. As complete observation and treatment of these numberless stars is out of the question, the plan is confined to a set of samples distributed over the whole of the sky.

Conclusion.

If, at the end of this lecture, somebody summarises what has been discussed by saying that the results about the structure of the universe are still very limited and not yet free from hypothetical elements, I feel little inclined to contradict him. But I would answer him by summing up in another way, viz.:—

Methods are not wanting which, given the necessary observational data obtainable in a moderate time, may lead us to a true, be it provisionally still not very detailed, insight into the real distribution of stars in space.

I think this time need not exceed some fifteen years. They to whom such a time may still seem somewhat long may be reminded of the fact that we shall have finished our work before any but a very few of our nearest neighbours in space can be aware of the fact that we have begun, even if we could send them a message now by wireless telegraphy travelling at the speed of light.

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

St. Andrews.—Besides the gifts of Diplodocus to the British Museum and to the museums of Paris and Berlin, Dr. Andrew Carnegie has, at the instigation of Dr. Holland, presented a neatly mounted example (cast) of the hind limb of Diplodocus to the University Museum, St. Andrews—another of the very munificent donations which mark the period of office of the late Rector of the University.